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Mixing time for a reciprocating plate agitator with flapping blades

S. Masiuk

Department of Chemical Engineering, Technical University of Szczecin, Al. Piastów 42, 71-065 Szczecin, Poland

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Abstract

The batch mixing of Newtonian liquids with reciprocating plate agitator was studied in a vessel of 0.248 m in diameter and of 0.678 m in height. A thermal-response technique was used for mixing time measurements. The mixing time characteristic was described by a modified dimensionless mixing time number — Reynolds number equation.

Keywords: Mixing; Reciprocating plate agitator; Mixing time

1. Introduction

This work is an extension of studies [1,2] made for the same reciprocating plate agitator with four flapping blades. The present study is an investigation of the effect of a wide range of process parameters on mixing time.

A modest amount of experimental data and no closed-form analytical solution are currently available for prediction of the mixing time in this case. The pioneering work in this field is due to Masiuk [3] who investigated the effect of process parameters on convective mixing time as a sum of time delay and time of transient response for a single perforated plate agitator in horizontal orientation working in the vertical direction. Sierzputowski and Oleniak [4] considered the simple model describing the concentration output response and they proposed expression representing the influence of pertinent variables on time delay and time constant numbers. Although the mixing time equation proposed in those papers is fair it gives only a crude information to the quality of mixing. More direct methods for determining mixing quality have been developed by measuring concentration or temperature within mixing vessel [5]. The thermal-response method was used for measuring mixing time by Masiuk et al. [6].

Although the view presented is certainly useful to explain the mixing process in particular with respect to dynamics of mixing [3,4] it is believed that unique dimensionless mixing time correlation for the reciprocating-plate agitator can encompass whole range of Reynolds number better than the earlier one proposed in the literature, where the dimensionless mixing time — Reynolds number characteristic, Nt∼Re, includes three different regions. In the laminar (creeping) and fully turbulent regions the Nt is independent of the Reynolds number and in the transient region the mixing time characteristic has a tendency to curve downward.

2. Theory

In the case of a constant viscosity the motion of homogenous viscous fluids is described by the Navier–Stokes equation of motion

$$\frac{\partial \rho \bar{v}}{\partial \tau} + \mathrm{div}(\rho \bar{v} \bar{v}) = \rho \bar{F} - \nabla p + \left(\kappa + \frac{1}{3}\mu\right) \nabla \,\mathrm{div}\, \bar{v} + \mu \,\Delta \bar{v} \quad (1)$$

When the density ρ is a constant (incompressible fluids, $\mathrm{div}\,\bar{v}=0$) the terms ∇p and $\rho \bar{F}$ can be combined if $\bar{F}$ is expressed as the gradient of a potential. When the dynamic pressure appearing there is small, term ∇p, may be omitted. If it is done, the general equation of motion (1) reduces to the simpler form

$$\rho \frac{\partial \bar{v}}{\partial \tau} + \rho \bar{v} \,\nabla \bar{v} = \mu \,\Delta \bar{v} \quad (2)$$

The second and third term in this equation represent two major mechanisms of mixing, namely convection and diffusion. The vector $\bar{v}$ is the mass average local velocity.

In the case of a reciprocating plate agitator the liquid flow is mainly parallel to the axis of the tank. The assumption which have been used is important restriction but it is believed that the general dimensionless equation of the present

E-mail address: smasiuk@carbon.tuniv.szczecin.pl (S. Masiuk)

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work is a compensation of applied simplification. According to this assumption, the Eq. (2) may be simplified to one-dimensional unsteady flow equation of motion

$$\rho\frac{\partial v}{\partial \tau}+\rho v\frac{\partial v}{\partial h}=\mu\frac{\partial^2 v}{\partial h^2} \tag{3}$$

The problem of one-dimensional fluid flow is widely scattered throughout engineering practice.

After substituting the non-dimensional quantities

$$\tau^*=\frac{\tau}{t};\qquad v^*=\frac{v}{v_{\max}};\qquad \rho^*=\frac{\rho}{\rho_{\mathrm{m}}};$$
$$\mu^*=\frac{\mu}{\mu_{\mathrm{m}}};\qquad h^*=\frac{h}{H} \tag{4}$$

into Eq. (3) the following non-dimensional equation is obtained

$$\frac{\rho_{\mathrm{m}}H^2}{t\mu_{\mathrm{m}}}\rho^*\frac{\partial v^*}{\partial \tau^*}+\frac{\rho_{\mathrm{m}}v_{\max}H}{\mu_{\mathrm{m}}}\rho^*v^*\frac{\partial v^*}{\partial h^*}=\mu^*\frac{\partial^2 v^*}{\partial h^{*2}} \tag{5}$$

In this equation the velocity $v_{\max}$ may be defined as the maximum displacement velocity which can be easily measured with standard equipment. The fluid properties ρ_{m} and μ_{m} are evaluated at the mean fluid temperature.

The Eq. (5) includes two dimensional groups characterising the mixing time problem. The first group is the dimensionless mixing time number Θ

$$\Theta=\frac{t\mu}{\rho H^2} \tag{6}$$

The second group may be transformed to the following form:

$$\frac{\rho_{\mathrm{m}}v_{\max}H}{\mu_{\mathrm{m}}}\frac{d_{\mathrm{h}}}{d_{\mathrm{h}}}=\frac{2\pi Afd_{\mathrm{h}}\rho_{\mathrm{m}}}{\mu_{\mathrm{m}}}\frac{H}{d_{\mathrm{h}}}=\mathrm{Re}\frac{H}{d_{\mathrm{h}}} \tag{7}$$

where Re is the Reynolds number. In the dimensionless groups Θ and Re the fluid parameters μ and ρ are used instead of μ_{m} and ρ_{m}. The Reynolds number is defined using the hydraulic diameter of the combined opening windows and annular gap. The hydraulic diameter d_{h} is calculated from the following relationships:

$$d_{\mathrm{h}}=\frac{4[(\pi/4)(D^2-d^2)+4ab]}{\pi(D+d)+8(a+b)} \tag{8}$$

Similar dimensionless mixing number is used in the relevant literature. Zlokarnik [7] has shown that the reduction mixing time ($t\mu/D^2\rho$) may be successfully used at an optimisation process of the rotational agitators. In this number a diameter of tank is used instead of the height of liquid H because the stream of the mixed liquid produced by rotational agitator mainly flow in radial direction from the blades to the cylindrical part of the tank.

An important consequence of this consideration is the relation between mixing time number and Reynolds number expressed by the following form:

$$\Theta=f\left(\mathrm{Re},\frac{H}{d_{\mathrm{h}}}\right) \tag{9}$$

3. Experimental details

The experimental system consisted of the mixer, associated electrical and thermal instrumentation, and the eccentric drive of the agitator. Fig. 1 is a diagram of the apparatus used in the mixing time tests. This apparatus was capable of producing thermal input and measurable variation in conductivity and temperature responses. The mixing was carried out with a single plate having four flapping blades [8]. The dimensions of the mixing vessel are given in Fig. 2, while the dimensions of the reciprocating plate agitator with flapping blades are presented in Fig. 3. The fast recorders MKT1 were used to record transient responses. The tap

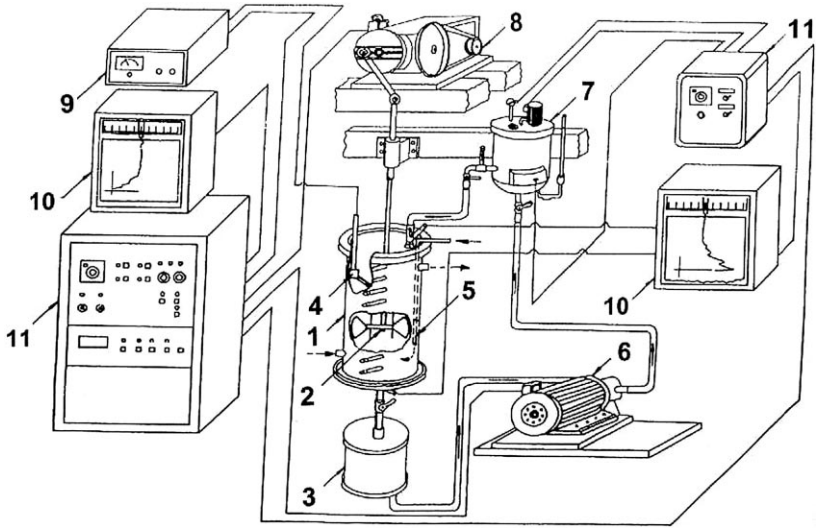

Fig. 1. Sketch of experimental set-up: (1) tank; (2) agitator; (3) storage tank; (4) conductance probe; (5) injection pipe; (6) circulating pump; (7) batchmeter; (8) drive; (9) conductometer; (10) MKT recorder and (11) power cubicle.

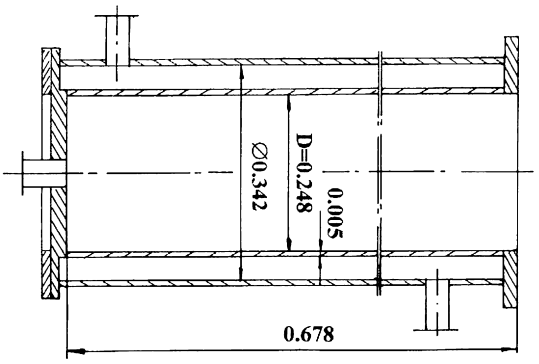

Fig. 2. Main dimensions of mixing vessel.

Table 1
Fluid properties

Liquid	ρ 20°C (kg/m^3)	μ 20°C (Pa s)
Aqueous solution of molasses		
Nr1	1415	6.95
Nr2	1384	2.45
Nr3	1371	2.21
Nr4	1353	0.27
Nr5	1337	0.15
Nr6	1317	0.13
Heavy oil	889	0.028
Transformer oil	860	0.016
Machine oil	867	0.01
Water	1000	0.001

water was used for the conductivity method. The conductance probe was placed at distance 0.06 m from the wall of the tank and 0.04 m under the surface of the liquid. In the experiments in all cases the height of liquid in the tank was 0.644 m giving the total liquid volume of ∼0.31 m^3. A saturated solution of NaCl (20 ml) was injected under the agitator near the place where the cylindrical part of the tank was connected with the bottom. In this method conductivity is higher after each test and the entire tank contents had to be emptied after three tests. Therefore, this is method of high operating costs and it is not applied for a high-priced fluids. The conductivity method was used in a few of the experimental tests. In the most part of the investigation the thermal-response technique was used. This method is very flexible and can be applied to large scale systems [5] but it requires complex measuring equipment. In this investigation this method is based on the principle of monitoring the change of temperature difference in two measuring point within mixed liquid. Temperatures were measured with bare iron–constantan thermocouples of 0.0015 m in diameter. One thermocouple was fixed 0.06 m under the liquid surface and 0.025 m from the wall of the tank. The second one was located 0.02 m above the bottom at the tank axis. Accordingly the thermocouples were connected in a differential circuit in order that, following addition of the hot liquid, the temperature difference between them could be recorded. An advantage of the thermal method is that it gives a unique defined quantitative value of the mixing time for the position measured. Aqueous solutions of molasses, heavy oil, transformer oil, machine oil and water were used in the experiments. The physical parameters of the fluids are given in Table 1. The thermal input signal was the well-mixed volume (0.004 m^3 at 80°C) of the same liquid as the main liquid was introduced into the cold liquid and with which it mixed. Before loading of the hot liquid the initial volume of the cold liquid in the tank was 0.027 m^3. In all experimental runs the total volume of the mixed liquid was always 0.031 m^3. Hot liquid feeder

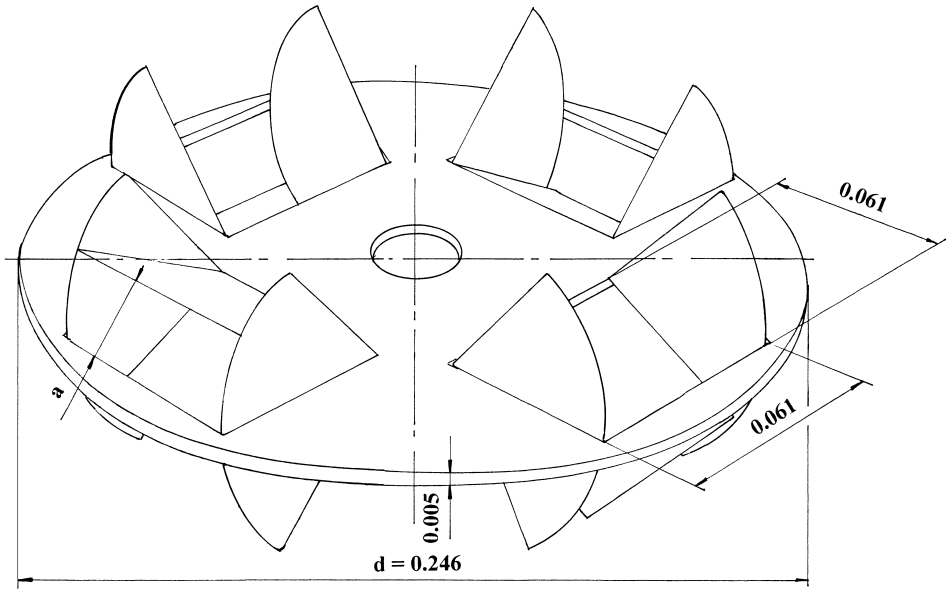

Fig. 3. Main dimensions of agitator.

was in the form of a short pipe of 0.04 m in diameter with the bottom which was fixed to the pipe by rods. There was a gap between the bottom and the rim of pipe through the hot liquid was distributed on the surface of cold liquid in the tank. This loading device was protected against premature penetration of cold bulk liquid in the tank of the mixer.

The convective mixing (exact blending) time experiments were conducted according to the following scheme:

1. Before the experimental measurements the bulk of the mixed liquid was cooled to the constant temperature by a cold stream of water flowing through the jacket of the tank. Cooling was conducted until the average temperature within the mixed liquid reached the values in the range from 18 to 22°C. Next the flow of cooling water was stopped.
2. The same liquid was heated in an outer batch meter to 80°C. A sample was withdrawn and added at the surface of cold bulk liquid with the agitator at rest. The addition time was about 3 s. The heat content of a sample was always the same.
3. After loading the sample the agitator and the temperature difference recorder were started simultaneously. The ΔT_{max} was of the order of 40°C.
4. During the measurement the cold liquid was pumped from the storage tank to the batch meter and new portion of the hot liquid was produced.
5. Working parameters of mixing process were changed and new thermal response was recorded. Figs. 4 and 5 illustrate typical examples of thermal response curves.

The mixing process was regarded as a complete when the average temperature within the mixed liquid did not change with time. The transient response curves are asymptotic to the time axis. This is an area of secondary steady state resulting from the mass or heat balances of the mixed liquid. It is difficult to detect precisely the end point of the transient process defining mixing time. In this work, this point is reached when the change in conductivity or the difference between the temperatures measured in two locations of the thermocouples remains smaller than ±1% of the overall increase of conductivity or temperature difference. The time between the beginning and the end of the transient process was defined as the mixing time. Using this procedure, however, it is only possible to find time which holds the degree of concentration or temperature homogeneity reached at the location actually measured and this time might be called the gross convective mixing time. Obviously, it is better to use more detectors and to observe reduction in a chosen statistical characteristic (variance, correlation functions) with time. In the case of this paper, it is difficult to realise this because the diameter of the agitator is slightly smaller than the

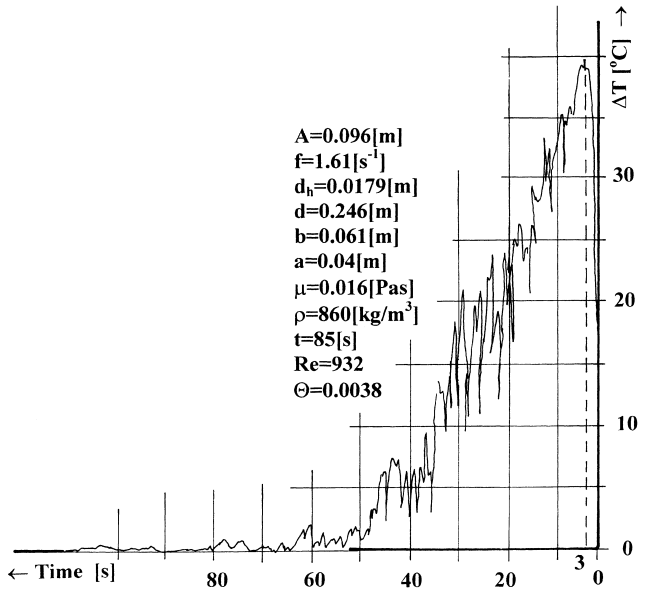

Fig. 5. Typical example of thermal response curve (d=0.246 m; Re=932).

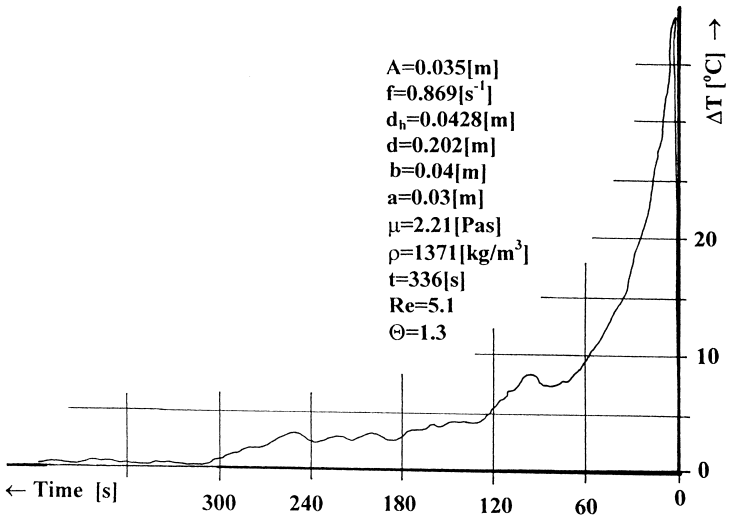

Fig. 4. Typical example of thermal response curve (d=0.202 m; Re=5.1).

diameter of the tank and moreover, almost half of the initial volume of the mixed liquid is penetrated by the moving agitator. This is a restricted area for location of the detecting elements.

4. Results

Accordingly to the relationship (9) plot of the data for a total of 367 runs obtained in this work for the effect of the Reynolds number is shown in Fig. 6. The conductivity data showed a greater experimental scatter than the data obtained using thermal method. This is a consequence of a form of conductivity response curves where the concentration achieved the secondary steady-state condition for which it is much more difficult to locate of a cutting point than the thermal response curves where the final difference of temperatures is equal to zero. Tabulated results for five liquids and different geometrical parameters of the agitator revealed the increase in dimensionless mixing time number. Furthermore, this increase is greater at a smaller Reynolds number than at a large Reynolds number. As follows from the Fig. 6 the mixing time characteristic as the $\Theta \sim \mathrm{Re}$ curve is very curvilinear dependence. The form of this characteristic is rather more analogous to the $\mathrm{Nt} \sim \mathrm{Re}$ curve for turbine and anchor agitators as it follows from the experimental results given by Hoogendoorn and den Hartog [9] and by Gramlich and Lamadé [10] for anchor agitator than to the form of similar relationship for screw agitator given by Novak and Rieger [11] or by Rieger et al. [12] and for anchor agitator by Zlokarnik [7]. Fournier et al. [13] estimated the micromixing time using turbine agitator. A system of parallel-competing reactions producing iodine was developed to study partial segregation. They stated that the micromixing time is directly proportional to $N^{-3/2}$. The published literature on mixing time experiments for rotational agitators is vast and it is not necessary to refer to all relevant studies. Only a few references have been cited where the authors gave the mixing time characteristic for the wide range of the Reynolds number and it is shown on Fig. 7.

Initially, the results given in Fig. 6 indicate a great reduction in the mixing time number with an increase in the Reynolds number. Up to a certain critical value of Reynolds number ($\mathrm{Re} < \sim 10$), i.e. in laminar region of flow, the experimental points follows a straight line with slope -2 and can be correlated in a standard form of relationship

$$\Theta = \text{constant} \times \mathrm{Re}^{-2}; \qquad \frac{H}{d_\mathrm{h}} = \text{constant} \tag{10}$$

As follows from the experimental points for the rotational agitators given by Hoogendoorn and den Hartog [9] the values of the Reynolds number exponent vary from -1.5 (marine propeller with draught tube) to -4 (three inclined-blade paddles without draught tube). The experimental results obtained by Lee et al. [14] working with turbine agitators were correlated in the relationship as mixing time versus the Reynolds number in which the Reynolds number exponent varied from -1.54 for $\mathrm{Re} < 60$ to -7.0 for $\mathrm{Re} > 60$. They stated that the transient region mixing time is smaller compared to laminar flow region mixing time. In most mixing time investigations the authors attention was concentrated on rotational agitators [11,15–19]. The analysis of their data indicate that the mixing time characteristics obtained in their tests are correlated for the low and height region of the

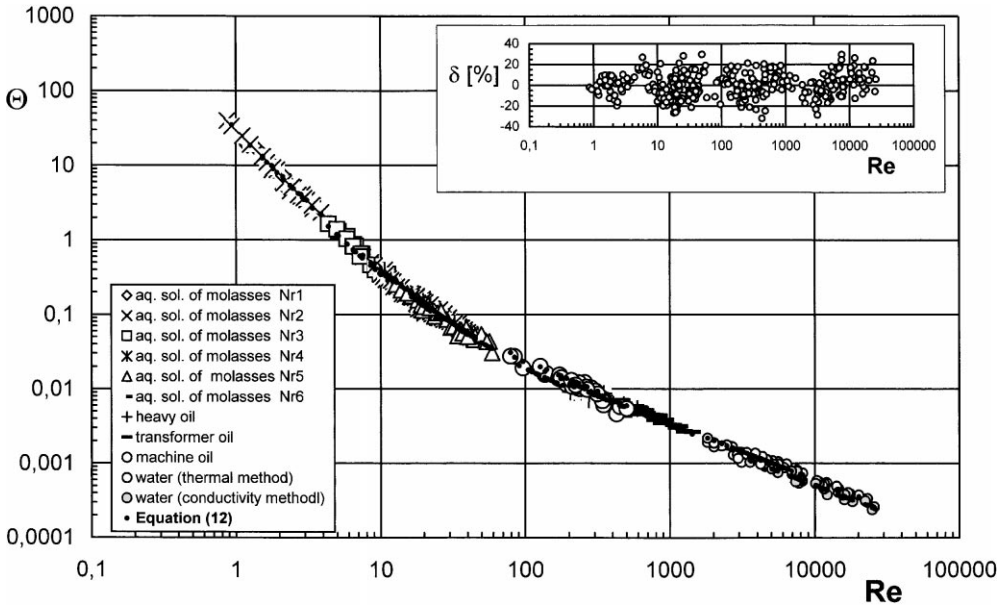

Fig. 6. Effect of Reynolds number on mixing time number.

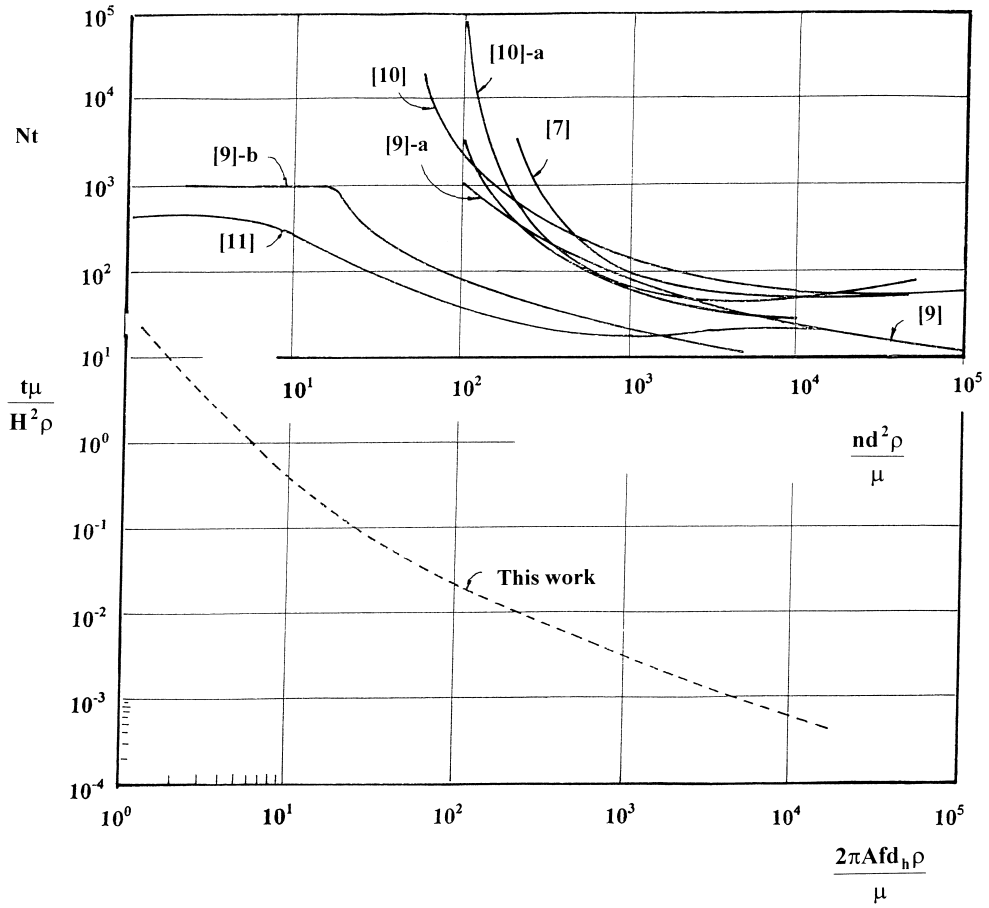

Fig. 7. Comparison of our results with those of other authors. Curves: [7] anchor; [9] anchor; [9] (a) turbine with baffles; [9] (b) marine propeller with draught tube; [10] anchor; [10] (a) turbine; [11] screw without baffles: dashed line — this study (d_h=0.01873 m).

Reynolds number and, therefore, it is asymptotic approximation on these limits. Their results of experiments are in conformity with roughly theoretical consideration of flowing fluid behaviour at least in principle. Their correlation cannot be compared with the present work where the experimental data approach a curve even in logarithmic system on the wide range of Reynolds number including the border regions.

However, there are considerable less studies in the literature [7,9–12] in which the authors presented the time mixing characteristics in the form of graphs, unfortunately, without analytical description. The data given in early mentioned literature are transformed into the solid lines and are shown in Fig. 7. The upper curves have been obtained for rotational agitators and are given in [7,9–11] without analytical description. The general trend of those curves is in an agreement with the results of present work. The dimensionless mixing time decreases sharply with increasing Reynolds number and considerably lower decreasing takes place at a region of large Reynolds number. Unfortunately, in this region their mixing time characteristics are different comparable to the curve obtained in this work when the mixing time decreases as long as the intensity of mixing increase inside of the bulk of the mixed liquid. With respect to the existing equations if a single dimensionless correlation is to be selected for all own data for both laminar and turbulent regions of flow it is believed to be more reliable.

In order to establish the effect of all the important general parameters on mixing time in wide range of variables data obtained in the present work have been analysed to propose the following relationship:

$$\Theta = C_1 \,\mathrm{Re}^A \left(\frac{H}{d_h}\right)^E (1 + C_2 \,\mathrm{Re}^B) \tag{11}$$

The constants and exponents are computed employing the principle of least squares and then Eq. (11) can be rewritten in the form

$$\Theta = 17 \,\mathrm{Re}^{-2} \left(\frac{H}{d_h}\right)^{3/20} (1 + 0.02 \,\mathrm{Re}^{5/4}) \tag{12}$$

This equation determines completely mixing time and it is strictly applicable over the range of the following experimental condition: A=0.025–0.096 (m); f=0.196–3.57

(s^{-1}); $2\pi A \cdot f$=0.08478–1.3082 (m/s); d=0.202 and 0.246 (m); a=0.012–0.078 (m); b=0.039 and 0.061 (m); t=55–9358 (s); d_h=0.006938–0.04839 (m); ρ=860–1415 (kg/m^3); μ=0.001–6.95 (Pa s); Re=0.87–2.6×10^3 and Θ=0.0002–40.

It is believed that these conditions cover the possible range of the operating and geometrical variables of interest in most practical situations. The main drawback of the Eq. (12) is that it does not include the effect of the number of plates installed on the shaft on the mixing time.

Fig. 6 presents a graphical form of Eq. (12) and additionally a plot of percentage error between measured and predicted values of Θ. The fit of data is remarkably good. The average percent error of all data is −0.104%. The standard deviation is σ=0.244. The difference between the prediction and measured is less than ±15% for approximately 80% of the data points.

Eq. (12), which is used to generalise of the experimental results of the present work, has somewhat different form than that commonly proposed to describe mixing time for rotational agitators. This equation is much more attractive because it describes the experimental data for the whole range of the Reynolds number using a continuous analytical function.

5. Conclusions

1. The proposed correlation (12) could be useful to practising process engineers and equipment designers.
2. The mixing time is described by the modified dimensionless mixing time number Eq. (6).
3. With respect to the very useful mixing time equations given in the literature which are singly obtained for rotary agitators at laminar and turbulent regions it would be interesting to see the uniform general correlation related to this problem. All in all, a single suitable equation will yield better conformity of experimental and theoretical consideration because the identification of distinct regimes flow cannot be exactly performed.

6. Nomenclature

a	clearance between plate and exhaust edge of flapping blade (m)
A	amplitude (m)
b	width of windows in the plate (m)
d	agitator diameter (m)
d_h	hydraulic diameter (m)
D	inner diameter of tank (m)
f	frequency (s^{-1})
F	external body force per unit mass (N)
h	spatial co-ordinate (m)
H	height of liquid level in the tank (m)
N	rate of agitator rotation (s^{-1})
p	pressure (Pa)
Re	Reynolds number
t	mixing time (s)
$\bar{v}$	velocity of fluid (m/s)

Greek letters

Θ	dimensionless mixing time number
κ	bulk coefficient of viscosity (Pa s)
μ	viscosity (Pa s)
ρ	density (kg/m^3)

Subscripts

h	hydraulic
m	mean
max	maximum
*	dimensionless

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